APPROXIMATE DETERMINATION OF TEMPERATURE FIELD IN HEAT-CARRIER FLOW WITH AXIAL HEAT CONDUCTION AND INTERNAL SOURCES

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The application is considered of the variational process to solve the axisymmetrical problem of convective heat exchange in which axial heat conduction and internal sources have been taken into account in laminar regime.

The heat-transfer condition for a stationary cylindrical axisymmetrical flow (Fig. 1) is given by

$$\lambda_0 \Delta T + q_v - c_p \gamma \overline{W} \nabla T = 0. \tag{1}$$

To be specific, the following boundary conditions are assumed:

$$T(x,\ R) = T_0 + (T_l - T_0)\frac{x^2}{l^2},\quad T(0,\ r) = T'_x(0,\ r) = 0. \tag{2}$$

The internal sources have the following distribution law:

$$q_v = q_0 w_x(x)\, w_r(r). \tag{3}$$

Since one only considers the laminar regime one has

$$T'_r = 0,\quad \nabla T = T'_x \bar{i},\quad \overline{W} = W_x \bar{i}. \tag{4}$$

It is assumed that the velocity distribution across the flow section follows the Hagen–Poiseuille law

$$W_x = 2W_0\left(1 - \frac{r^2}{R^2}\right). \tag{5}$$

By introducing the notation

$$t = \frac{T - T_0}{T_l - T_0},\quad \rho = \frac{r}{R},\quad \psi = \frac{x}{R}, \tag{6}$$

$$q = \frac{q_0 R^2}{\lambda_0 (T_l - T_0)},\quad k = \frac{R^2}{l^2}, \tag{7}$$

one obtains the original heat problem transformed into

$$L \equiv \Delta t + q w_\rho w_\psi - 2\,\mathrm{Pe}\,(1 - \rho^2)\, t'_\psi = 0, \tag{8}$$

$$t(\psi,\ 1) = k\psi^2,\ t(0,\ \rho) = t'_\psi(0,\ \rho) = 0. \tag{9}$$

One now introduces a system u_i of coordinate functions and attempts to find an approximate solution by using the Kantorovich variational process within the class of functions which satisfy the boundary conditions (9), namely

$$\bar{t} = k\rho^2\psi^2 + \sum_i u_i(\rho)\, s_i(\psi). \tag{10}$$

The introduction of the term $k\rho^2\psi^2$ enables one to determine the minimizing functions s_i for zero boundary conditions.

If in Eq. (8) $\bar{t}$ is inserted one obtains

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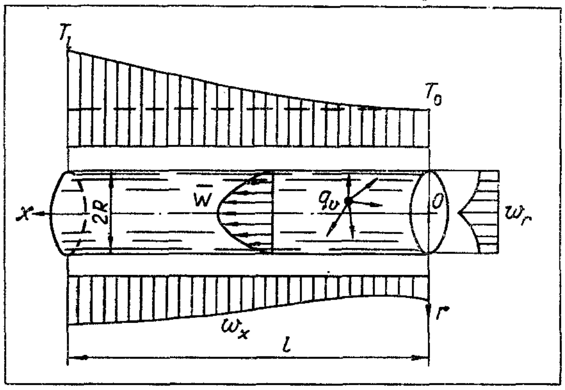

Fig. 1. Computation diagram of cylindrical flow.

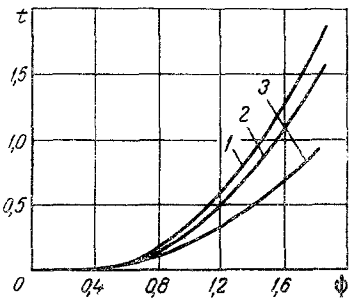

Fig. 2. Graph of axial distribution of temperature in the flow: 1) $\rho = 0$; 2) 0.5; 3) 1.0.

$$\overline{L} \equiv \sum_i \left[u_i s_i'' + \frac{1}{\rho} (\rho u_i')' s_i - 2\,\mathrm{Pe}\,(1-\rho^2)\, u_i s_i' \right] + q w_\rho w_\psi + 4k\psi^2 + 2k\rho^2 - 4\mathrm{Pe}\,(1-\rho^2) k\rho^2\psi = 0. \tag{11}$$

A simple problem is now solved:

$$(\rho u')' + q\rho w_\psi w_\rho = 0, \tag{12}$$

$$u'(0) = 0, \quad u(1) = 0. \tag{13}$$

In view of the boundary conditions (13) the solution can be written as

$$u = q w_\psi [\Phi(1) - \Phi(\rho)], \quad \Phi(\rho) = \int \frac{1}{\rho} \int w_\rho \rho d\rho. \tag{14}$$

It is not difficult to see that if one includes qw_ψ in s_i and constructs u_i in the form

$$u_i = u^i, \tag{15}$$

then the conditions (13) are satisfied for any i; the system (15) is complete in the region $0 \le \rho \le 1$ and it can thus be considered as a coordinate system.

Equation (8) is nonself-adjoint; therefore, there is no energy functional corresponding exactly to it. However, a functional can be found which satisfies (8) approximately. For zero boundary conditions the Euler equations for the minimizing functions can be written as

$$\int^1 \overline{L u_k} \rho d\rho = 0. \tag{16}$$

Nondimensional coefficients are now introduced:

$$a_{ik} = \int_0^1 u_i u_k \rho d\rho, \quad c_k = \int_0^1 w_\rho u_k \rho d\rho, \quad b_{ik} = \int_0^1 u_i' u_j' \rho d\rho,$$
$$l_{ik} = \int_0^1 2\mathrm{Pe}\,(1-\rho^2)\, u_i u_k \rho d\rho, \quad d_k = \int_0^1 2k\rho^3 u_k d\rho, \tag{17}$$
$$h_k = \int_0^1 4\mathrm{Pe}\,(1-\rho^2)\, k\rho^3 u_k d\rho, \quad f_k = \int_0^1 u_k k\rho d\rho.$$

Then by integrating (16) with respect to ρ one obtains

$$\sum_i \nu_{ki} s_i = \varphi_k, \tag{18}$$

where $\nu_{ki} = a_{ki} d^2/d\psi^2 - l_{ki} d/d\psi - b_{ki}$ is a differential operator; $\varphi_k(\psi) = h_k\psi - d_k - qc_k w_\psi - f_k\psi^2$ is a known function.

To give an example of the implementation of the described variational process, the temperature field was determined in a flow of liquid-metal heat exchanger (Fig. 2).

NOTATION

T	is the temperature;
x and r	are the axial and radial coordinates;
λ	is the thermal conductivity;
c_p	is the specific heat flux;
γ	is the specific weight;
W	is the velocity;
q	is the power of internal sources;
Pe	is the Péclet number;
ψ and ρ	are the axial and radial dimensionless coordinates;
t	is the dimensionless function of temperature;
s	is the minimizing function;
u	is the coordinate function;
L and ν	are the differential operators.

Subscripts

0	initial;
l	final;
x and ψ	axial;
r and ρ	radial;
i	current.

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